Geostrophic balance

OCN620

Geostrophic balance

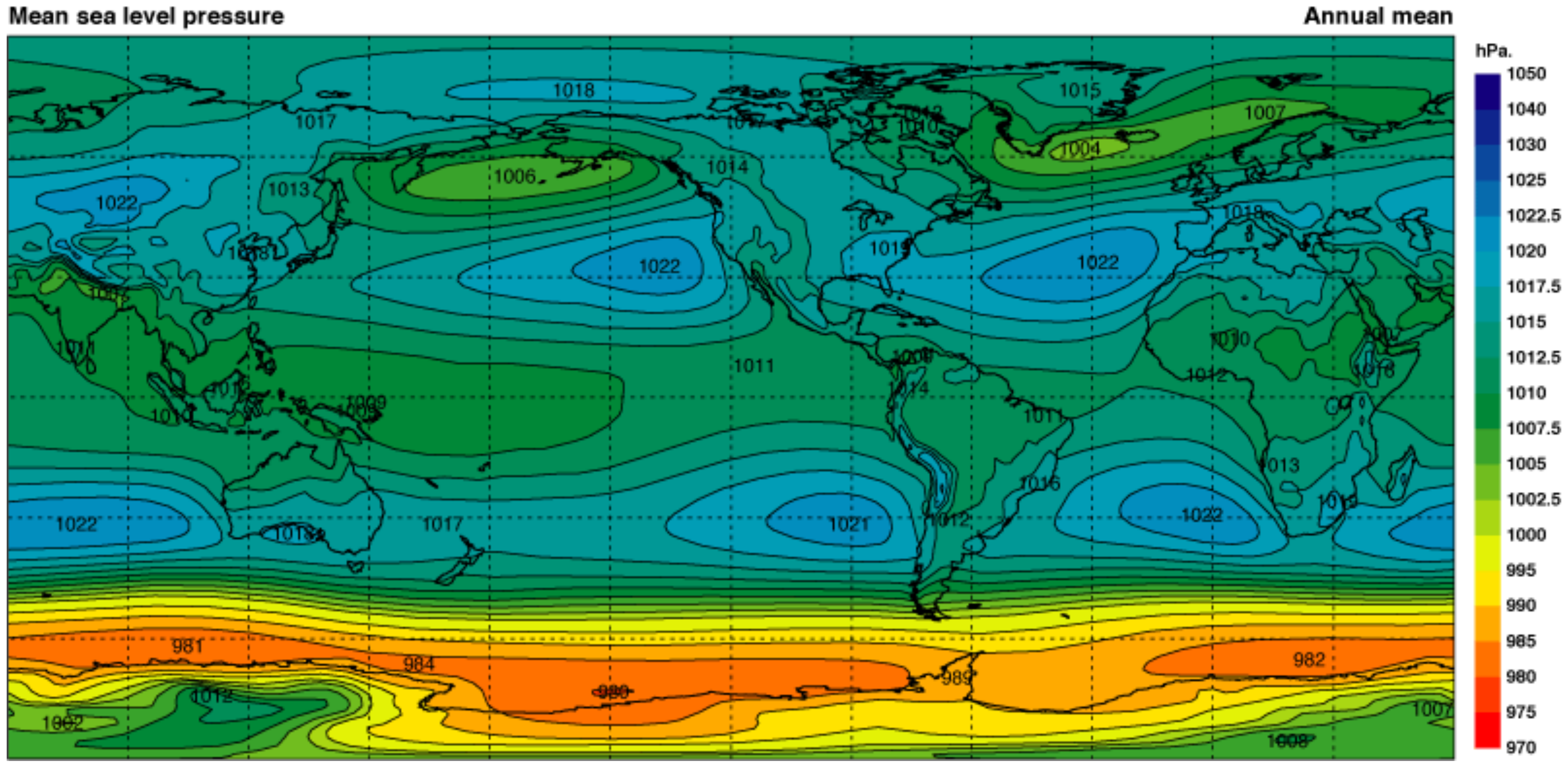
Key concepts

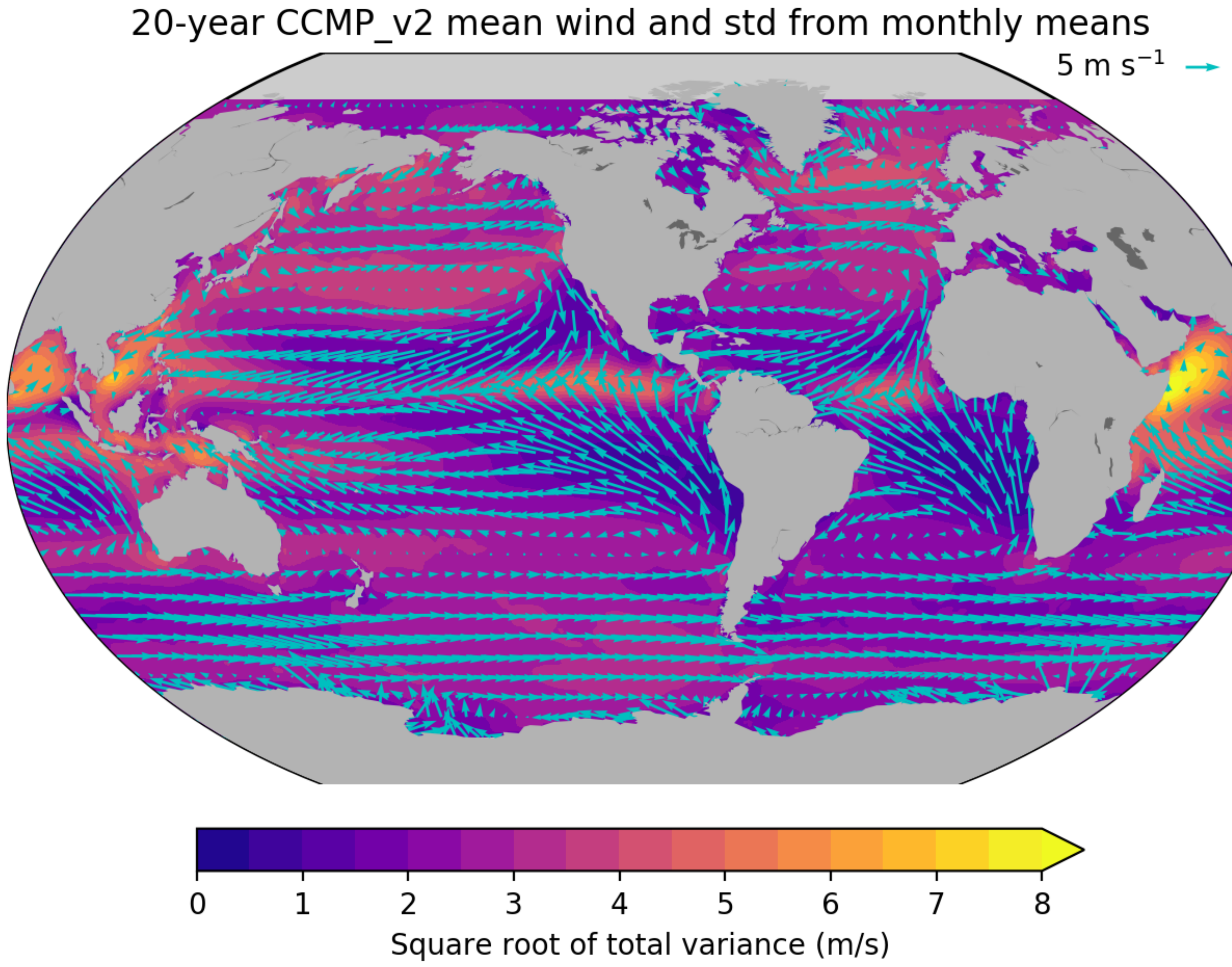
- For large-scale flows away from frictional boundaries, **steady** pressure gradient force is balanced by **steady** geostrophic flow.
- Geostrophic flow is 90º to the right of the pressure gradient force in the Northern Hemisphere; 90º to the left in the Southern Hemisphere.
- Hydrostatic balance organizes most of the ocean into stratified layers.
- Rotation causes horizontal flow within layers of constant density to be vertically uniform.

Flow along lines of constant pressure

Surface winds

Credit: ECMWF, ERA-40 Atlas

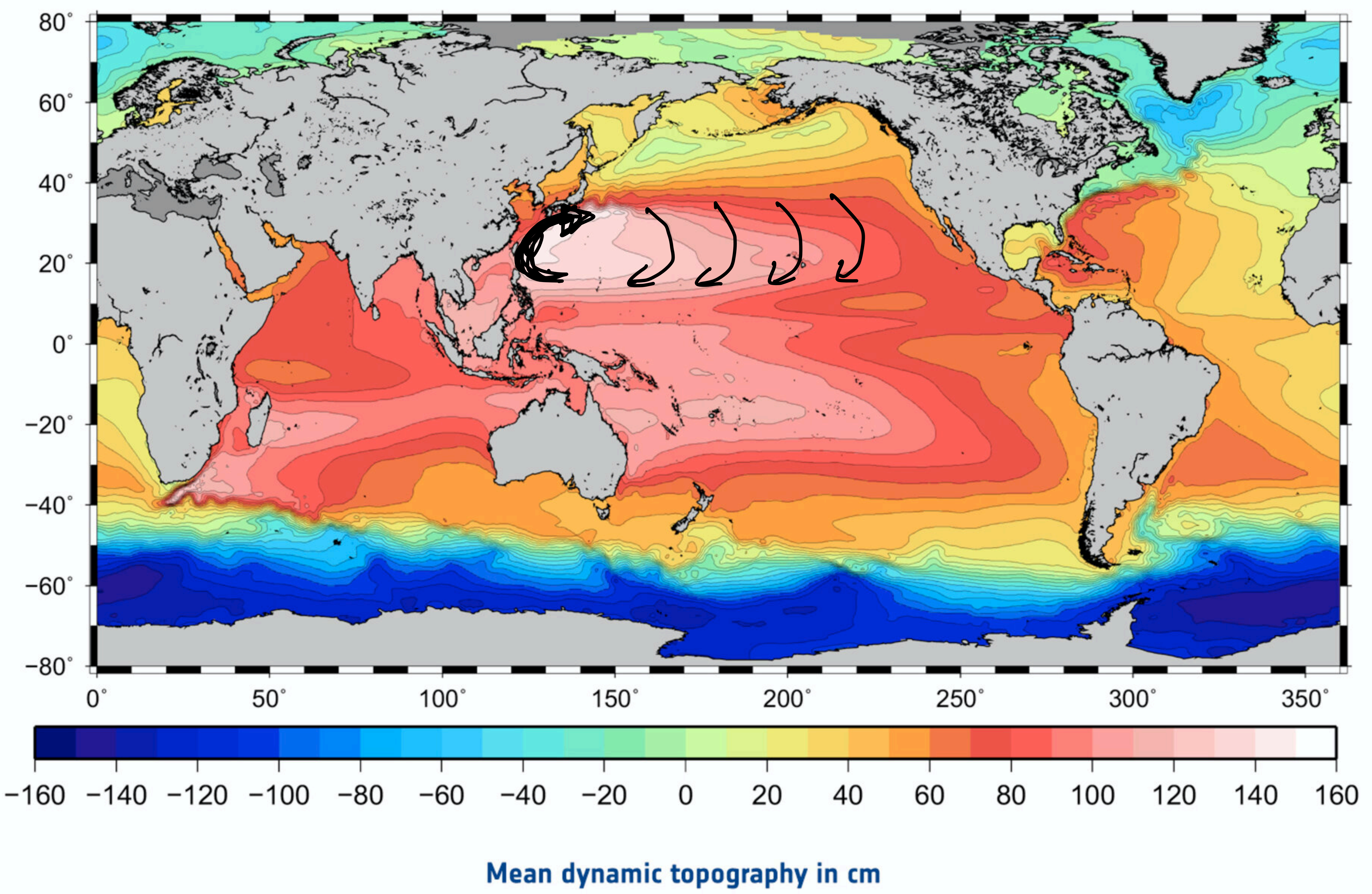




Flow along lines of constant pressure

Surface currents

- Mean dynamic topography (MDT) is analogous to the atmospheric pressure field.
- Calculated as mean sea surface height from satellite altimetry minus the geoid.
- Flow is along lines of constant pressure (i.e., height).
- Representative of mean geostrophic velocity.



Equations of motion

Review

- In many cases, physical oceanographers use **simplified** versions of the full equations of motion by making **appropriate approximations**.
    - This allows us to extract the “big picture” from the complexity and identify fundamental **balances**.
- In order to understand circulation at the space and time scales of ocean eddies and larger (including subtropical gyres), we make the following approximations:
    1. Incompressibility
    2. Local Cartesian plane (not a spherical Earth)
    3. Hydrostatic balance (vertical acceleration is not important)
    4. Boussinesq approximation (density variations are not important unless related to buoyancy)
    5. Friction is not important away from boundary layers.

Steady-state, hydrostatic, and Boussinesq

Simplified and widely applicable momentum equations

- In component form:

$$\hat{\imath}: \quad -fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{1}{\rho_0}\frac{\partial \tau^x}{\partial z}$$

$$\hat{\jmath}: \quad fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \frac{1}{\rho_0}\frac{\partial \tau^y}{\partial z}$$

$$\hat{k}: \quad -g\rho = \frac{\partial p}{\partial z}$$

- Equations of motion also include continuity:

$$\nabla \cdot \overrightarrow{u} = 0$$

Remember $w \neq 0$.

Ocean interior vs. frictional boundaries

Geostrophic vs. ageostrophic flow

- Split the velocity into geostrophic and ageostrophic (i.e., Ekman) components,

$$\vec{u} = \vec{u}_g + \vec{u}_e$$

- For the zonal component,

$$\hat{\imath}: \quad -fv = -f(v_g + v_e) = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{1}{\rho_0}\frac{\partial \tau^x}{\partial z}$$

- Geostrophic balance everywhere;
  ageostrophic and geostrophic in frictional boundaries.

Geostrophic balance:

$$-fv_g = -\frac{1}{\rho_0}\frac{\partial p}{\partial x}$$

Ageostrophic balance:

$$-fv_e = \frac{1}{\rho_0}\frac{\partial \tau^x}{\partial z}$$

Geostrophic balance

Steady balance between pressure gradient and Coriolis

- In component form:

$$\hat{\imath}: \quad fv_g = \frac{1}{\rho_0}\frac{\partial p}{\partial x}$$

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- Implications:
  - Rotating frame: steady forcing → large-scale steady geostrophic flow.
    - Inertial frame: steady-forcing → acceleration.
  - Flow tends to be vertically uniform in a fluid (or layer of fluid) with constant density.
    - E.g., Taylor columns, which we will explore in our next tank demonstration.

Geostrophic balance

Steady balance between pressure gradient and Coriolis

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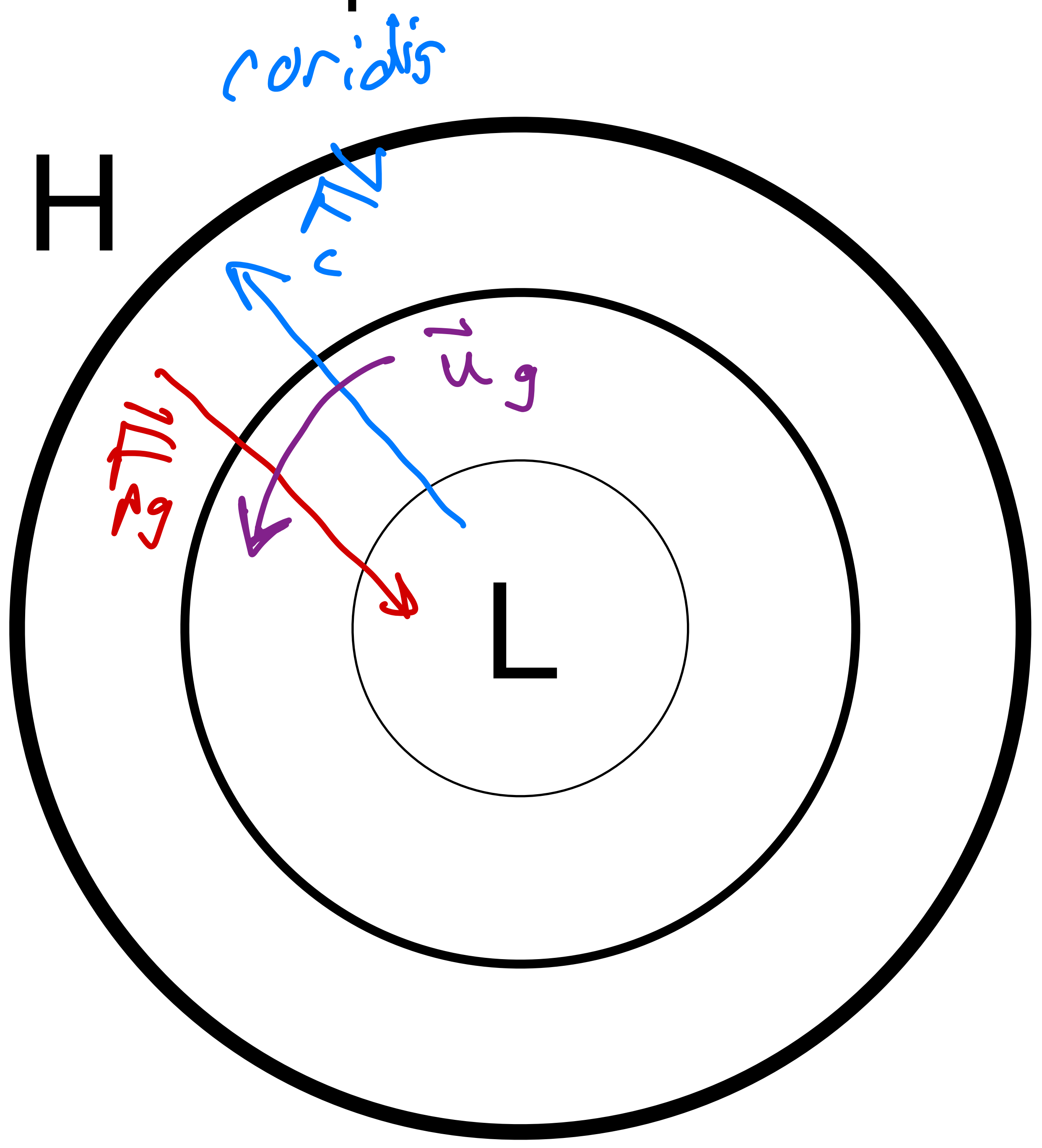
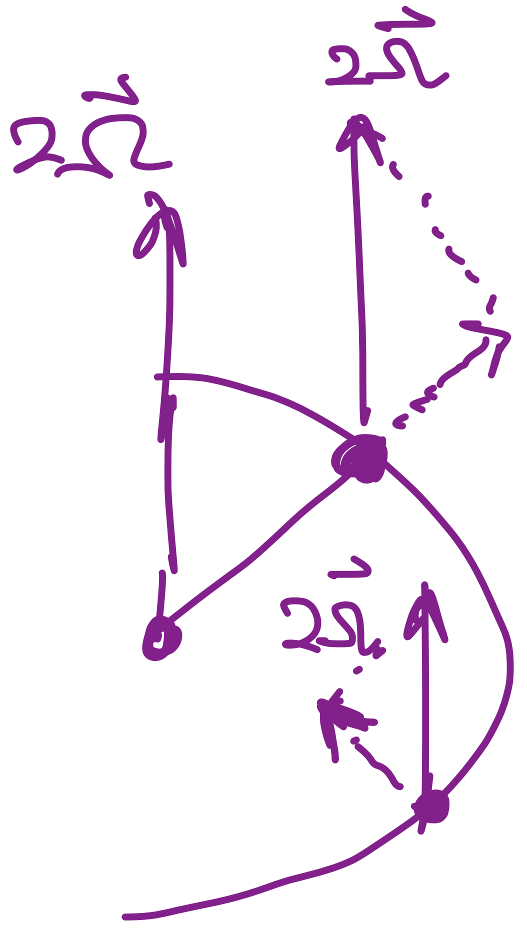
- Northern Hemisphere:



$f\hat{k} = 2\vec{\Omega}\sin\theta$



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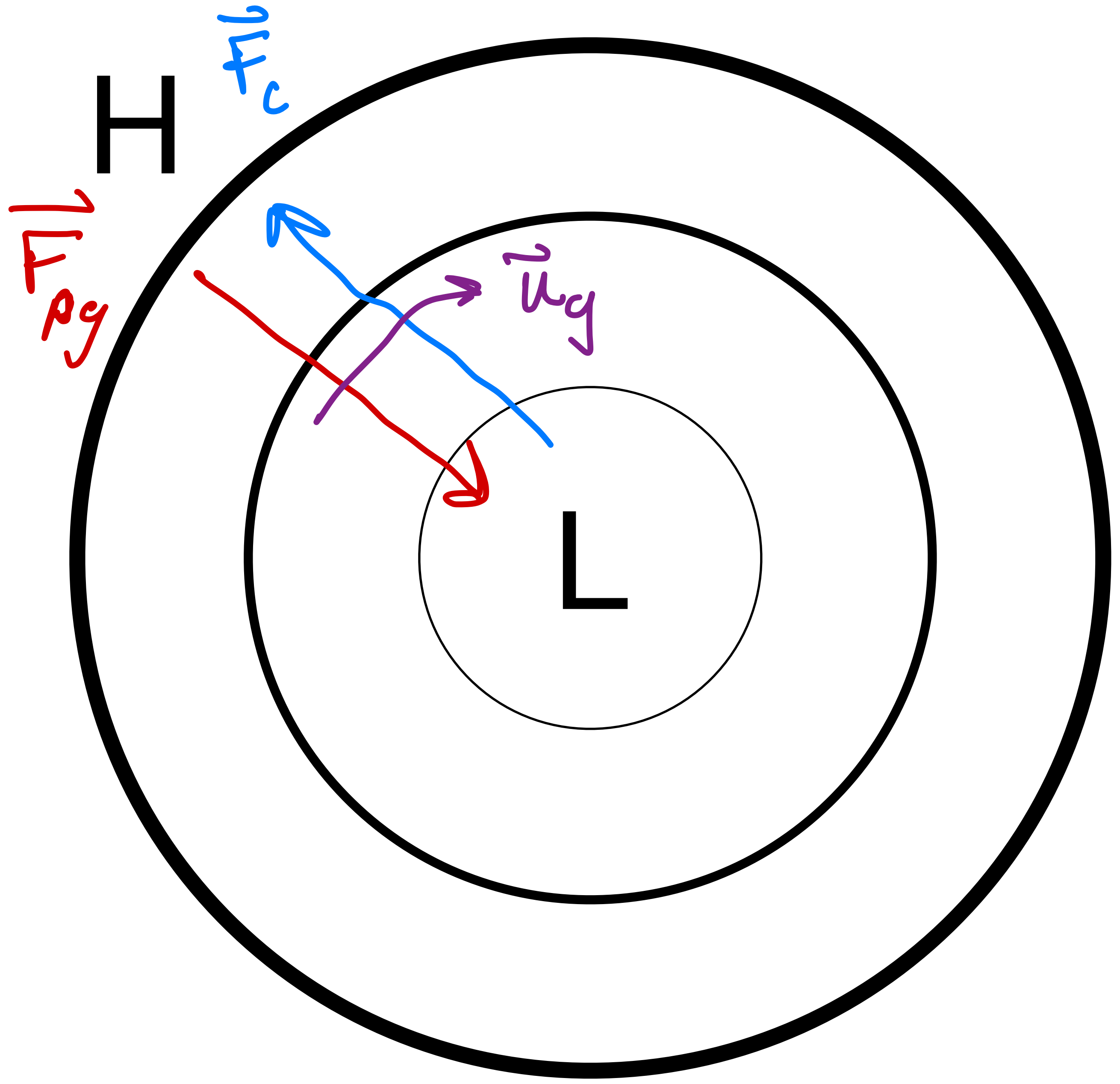
- Southern Hemisphere:



Geostrophic balance

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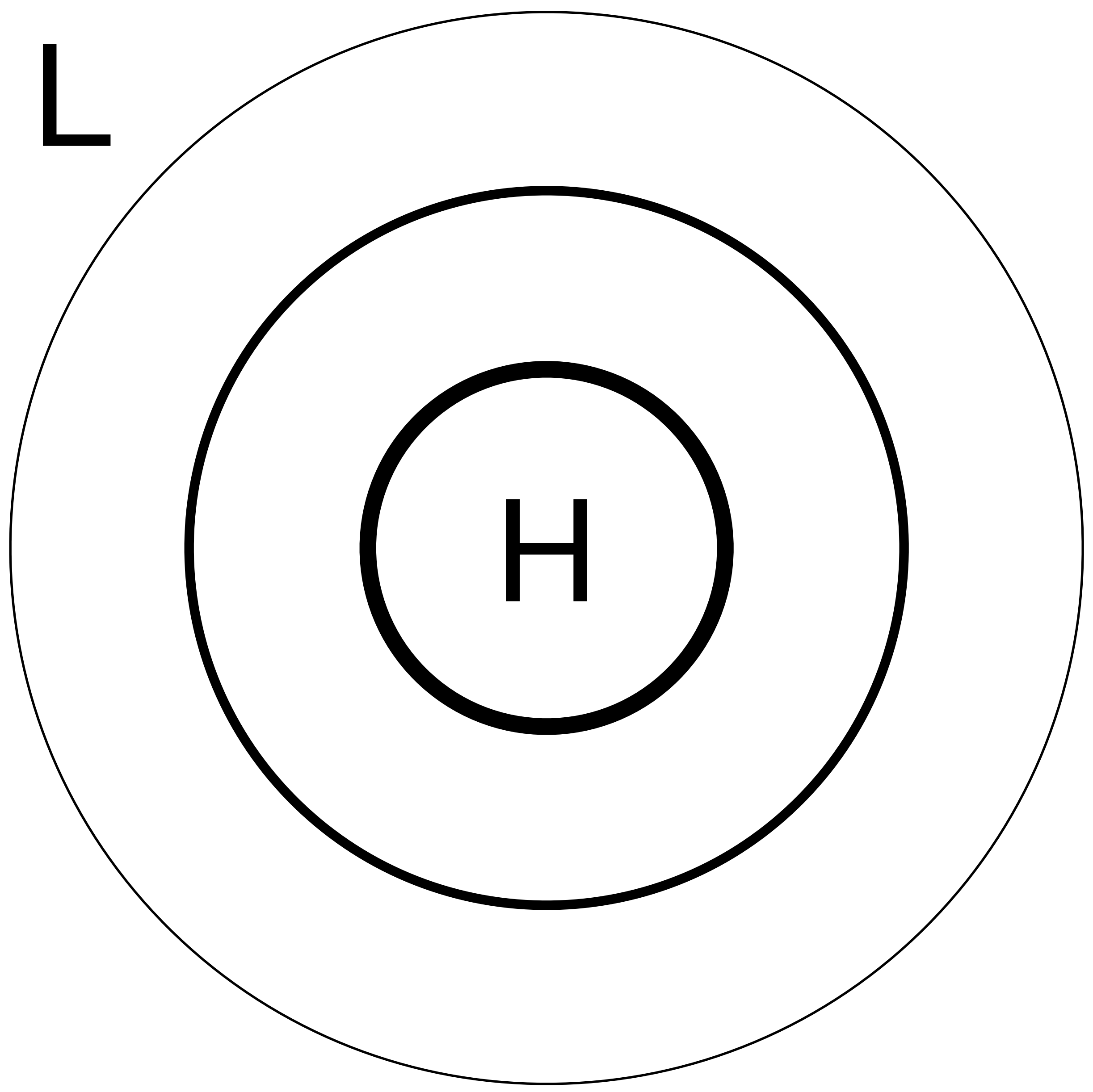
- Northern Hemisphere:



Geostrophic balance

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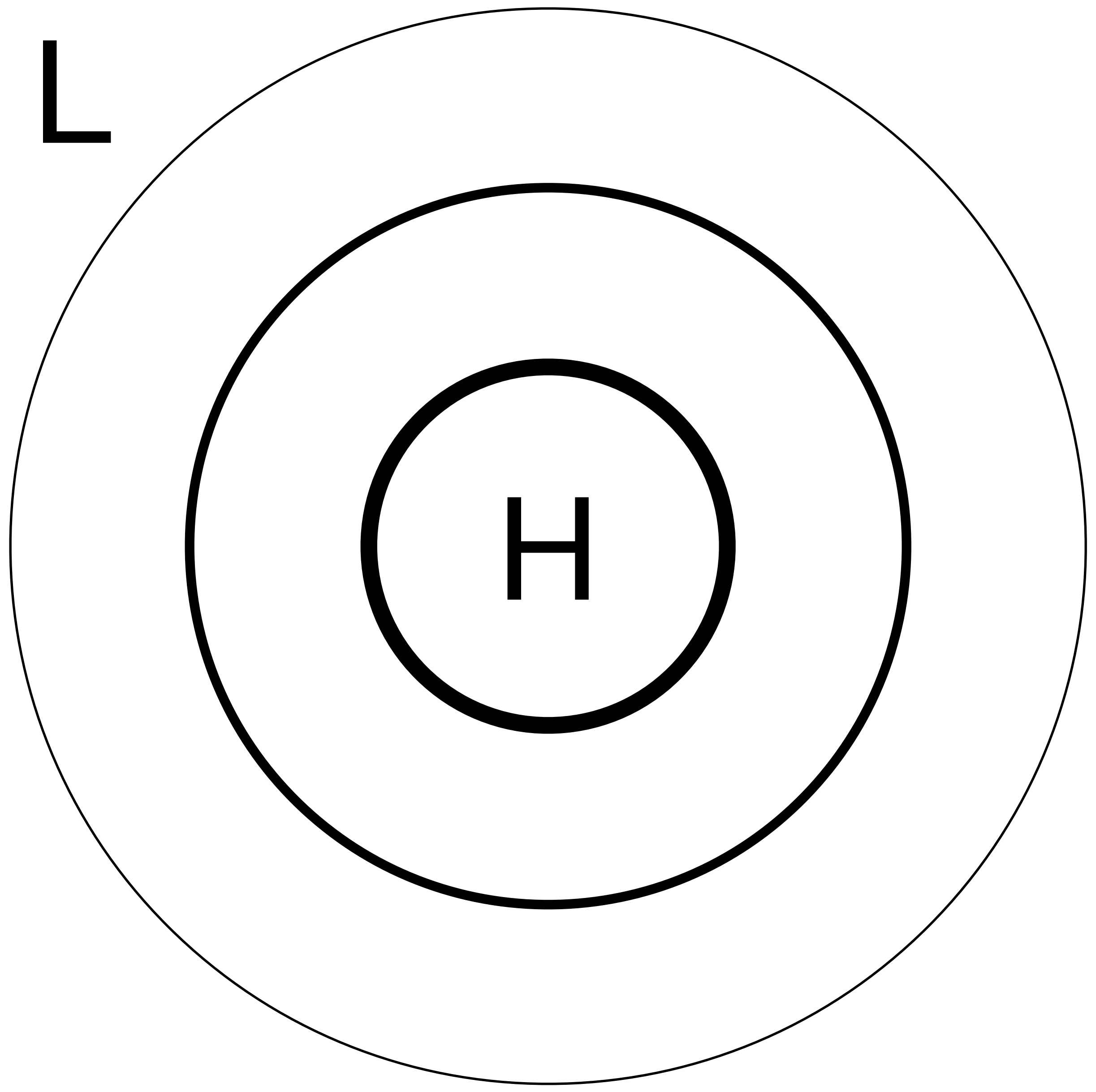
- Southern Hemisphere:



Geostrophic balance

Surface geostrophy and sea surface height

- In component form:

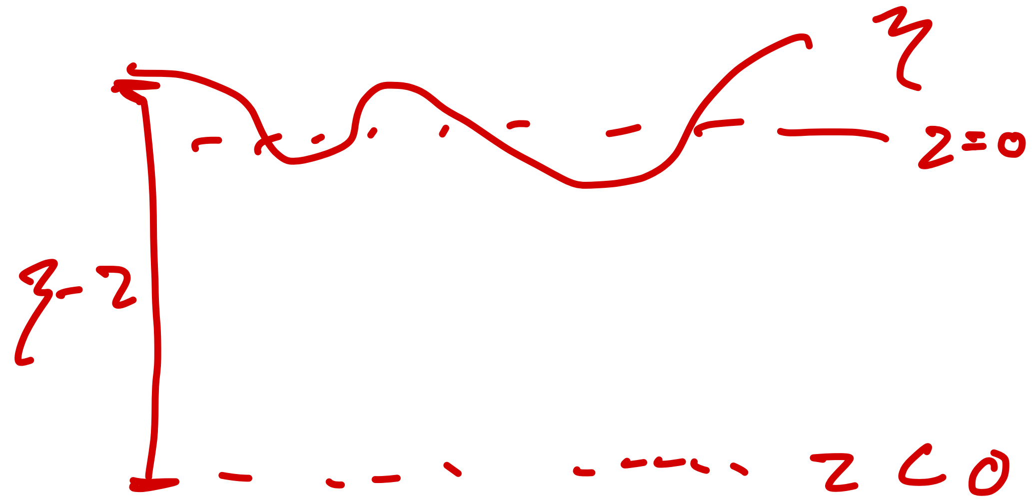
$$\hat{\imath}: \quad f v_g = \frac{1}{\rho_0}\frac{\partial p}{\partial x}$$

$$\hat{\jmath}: \quad f u_g = -\frac{1}{\rho_0}\frac{\partial p}{\partial y}$$

$$\hat{k}: \quad -g\rho = \frac{\partial p}{\partial z}$$

- In the surface mixed layer with constant density, **vertically integrate hydrostatic balance** and write pressure in terms of sea surface height:

$$p(z) = g\rho_0(\eta - z)$$



$$\frac{1}{\rho_0}\frac{\partial p}{\partial x} = \frac{1}{\rho_0}\left[g\rho_0\left(\frac{\partial \eta}{\partial x} - \frac{\partial z}{\partial x}\right)\right]$$

0

$$v_g = \frac{g}{f}\frac{\partial \eta}{\partial x}$$ ← surface height

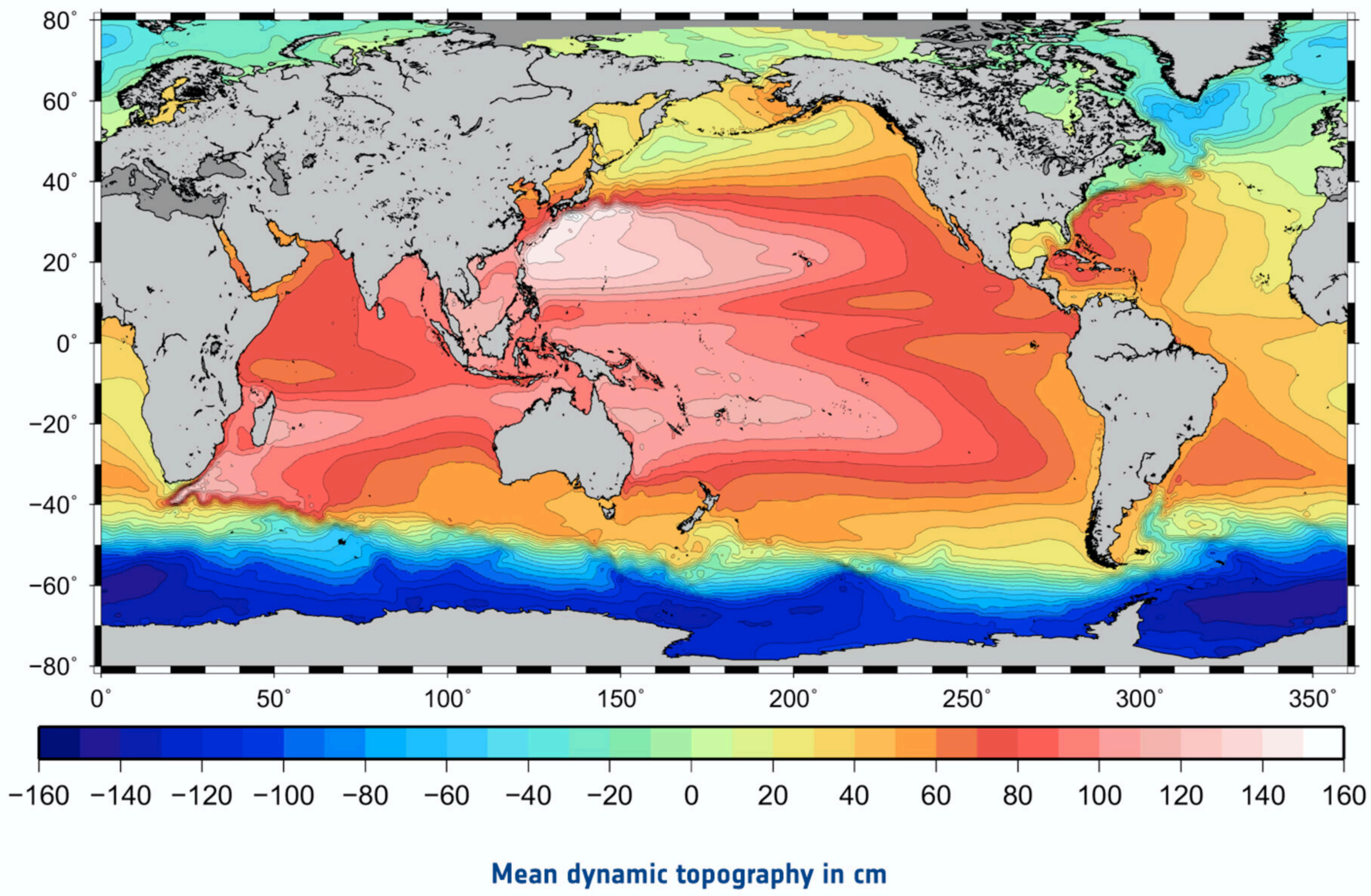
Geostrophic balance

Surface geostrophy and sea surface height

- In component form:

$$\hat{i}: \quad v_g = \frac{g}{f}\frac{\partial \eta}{\partial x}$$

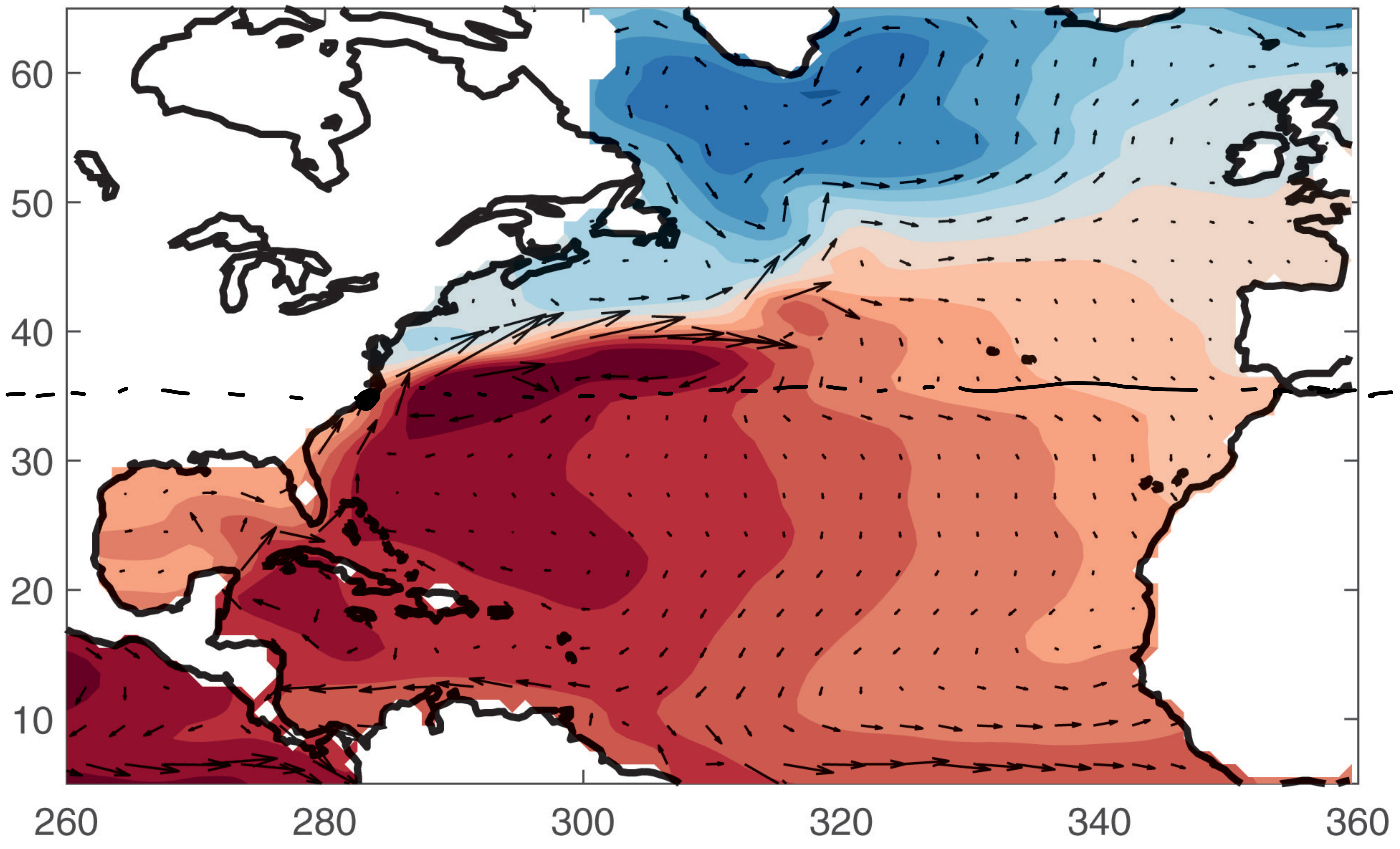
$$\hat{j}: \quad u_g = -\frac{g}{f}\frac{\partial \eta}{\partial y}$$



Geostrophic balance

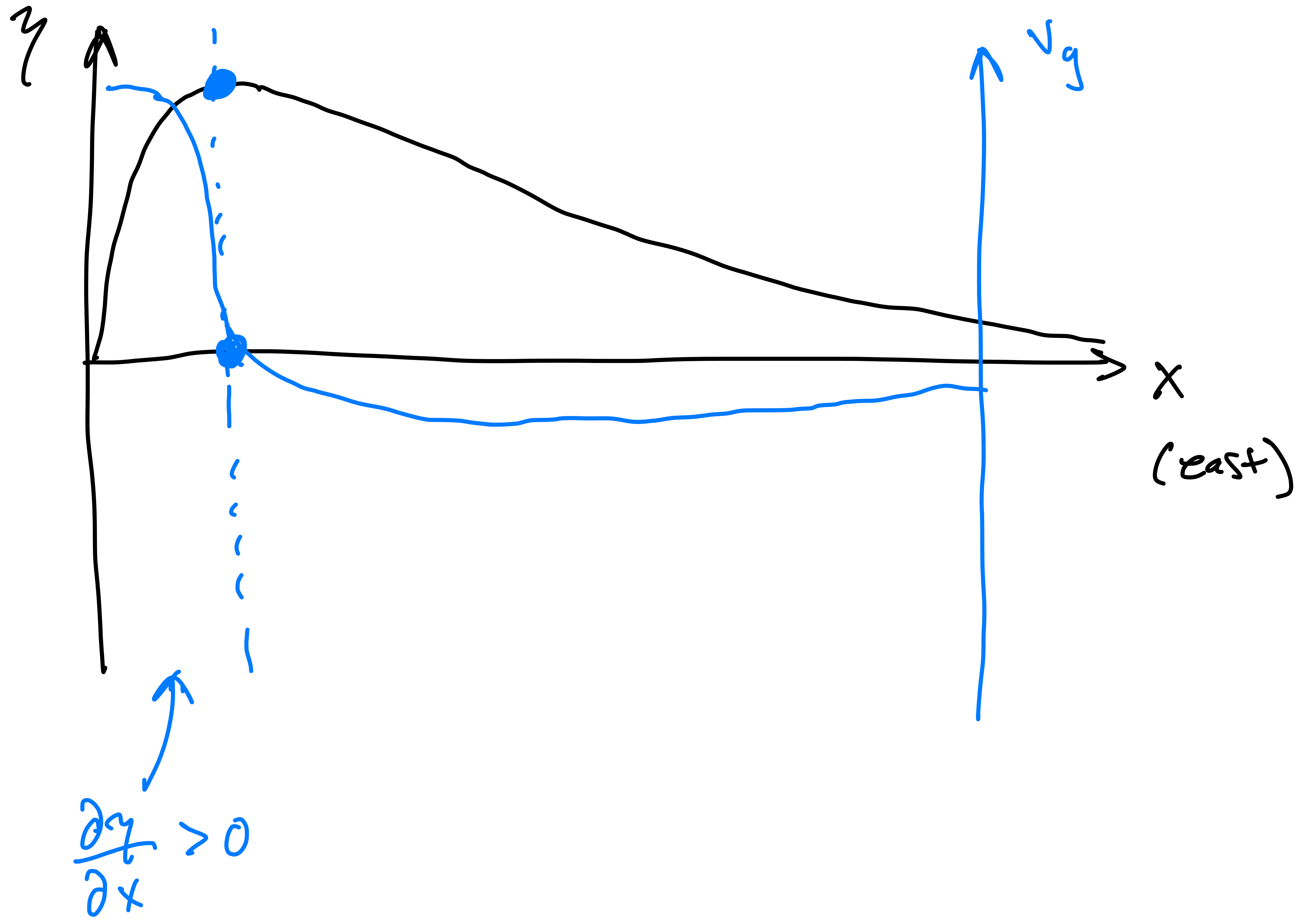
Example: North Atlantic

$$\hat{\imath}: \quad v_g = \frac{g}{f}\frac{\partial \eta}{\partial x} \qquad \hat{\jmath}: \quad u_g = -\frac{g}{f}\frac{\partial \eta}{\partial y}$$



Sea surface height above geoid η averaged from 1993 to 2010 from the 1° AVISO satellite altimeter product with η contour interval 0.1 m and values spanning roughly –0.6 to 0.9 m. From Karnauskas (2020), Figure 4.6.

- Zonal sections of η and v_g: 35° N



Geostrophic balance

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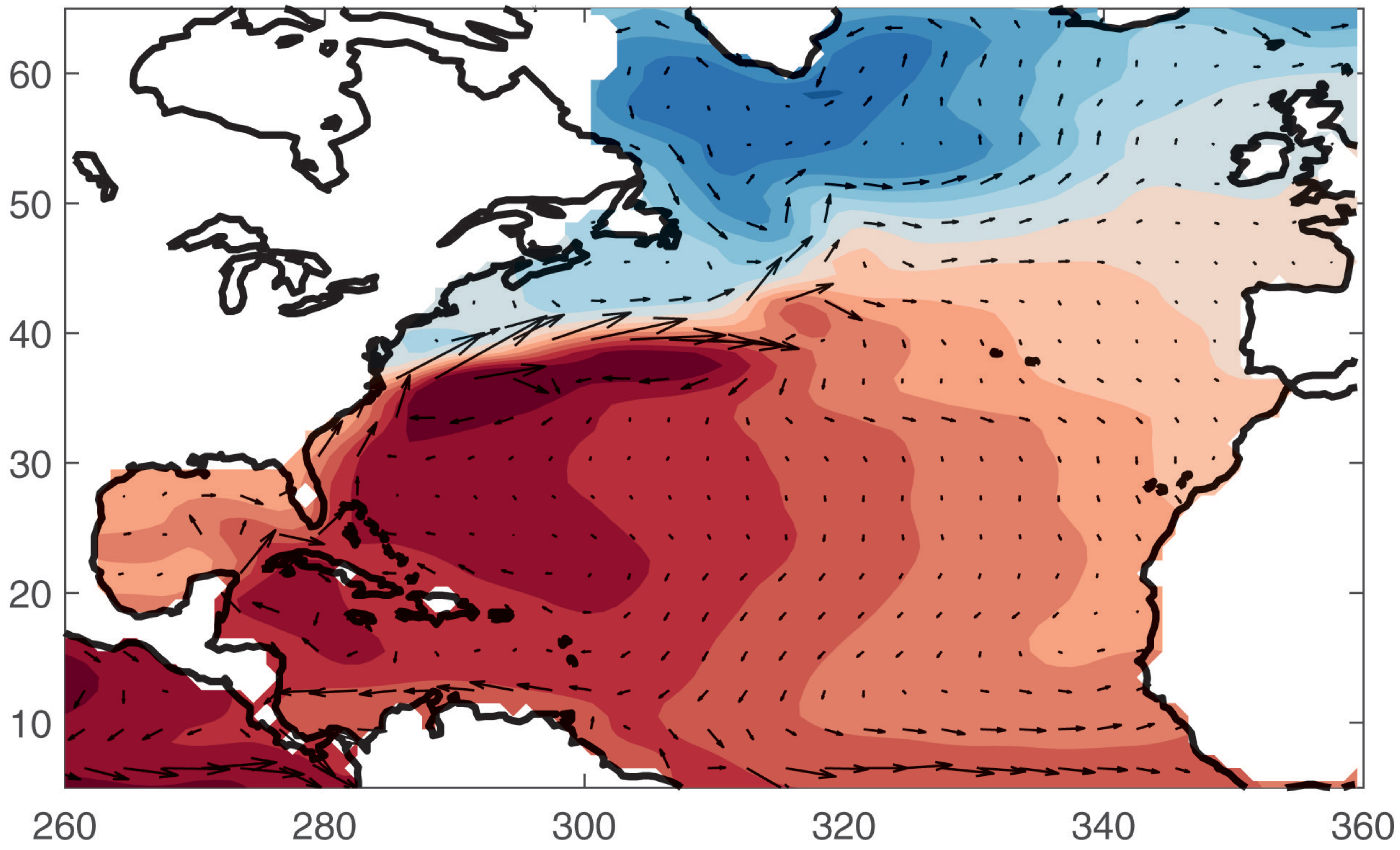


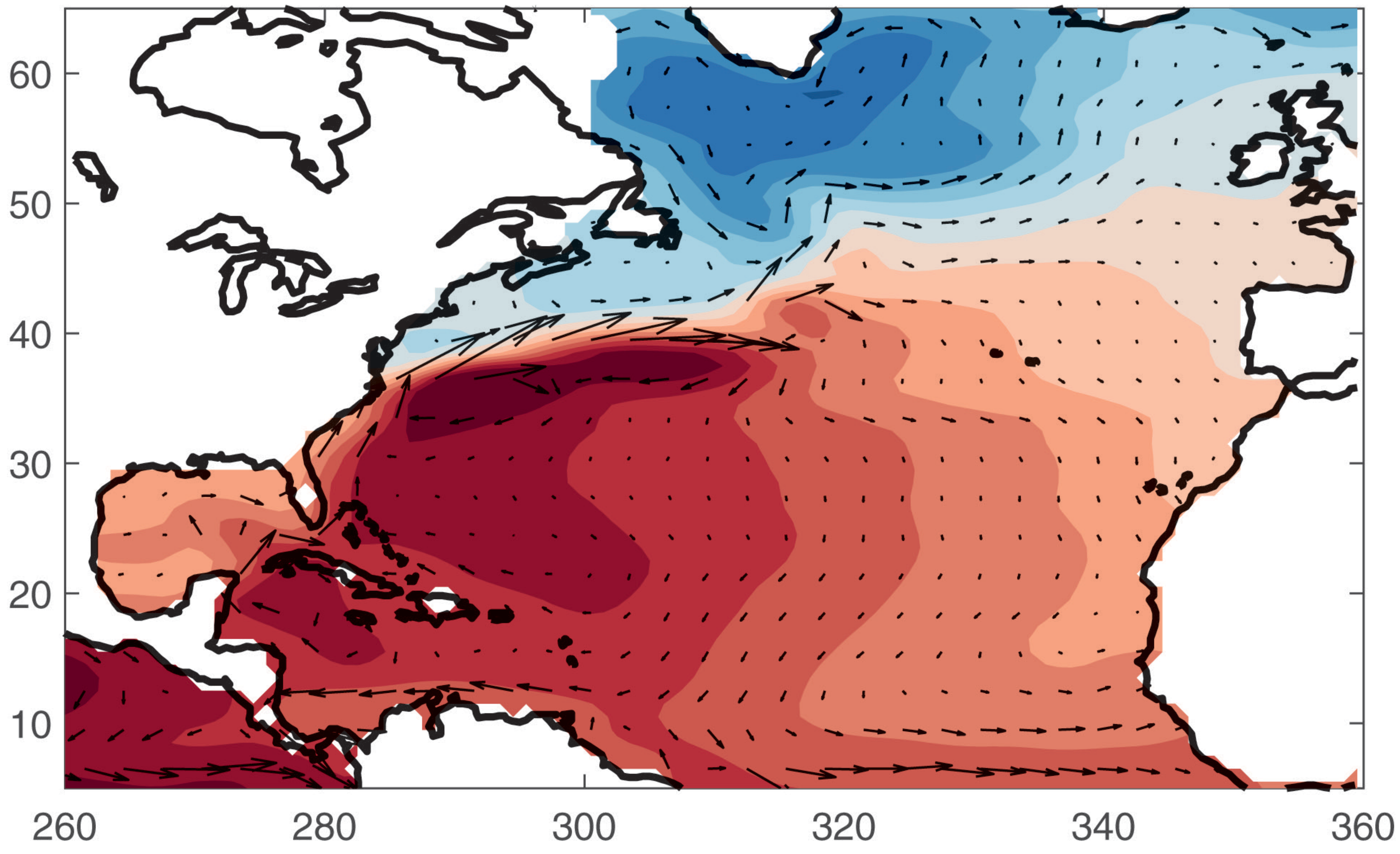
Sea surface height above geoid η averaged from 1993 to 2010 from the 1° AVISO satellite altimeter product with η contour interval 0.1 m and values spanning roughly –0.6 to 0.9 m. From Karnauskas (2020), Figure 4.6.

- Meridional sections of η and u_g: 300°E

Geostrophic balance

Example: North Atlantic

$$\hat{\imath}: \quad v_g = \frac{g}{f}\frac{\partial \eta}{\partial x} \qquad \hat{\jmath}: \quad u_g = -\frac{g}{f}\frac{\partial \eta}{\partial y}$$



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- Estimate surface velocity of the North Atlantic current:

$$u_g \sim -\frac{g}{f}\frac{\Delta \eta}{\Delta y}$$

$10^{-4}\,\frac{1}{s}$

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Geostrophy + constant density

Vertically uniform geostrophic flow

- Hydrostatic balance:

$$\hat{k}: \quad -g\rho \;=\; \frac{\partial p}{\partial z}$$

- If ρ is constant, vertical pressure gradient is constant:

$$\hat{k}: \quad \text{constant} \;=\; \frac{\partial p}{\partial z}$$

- Pressure changes with depth at the same rate everywhere.

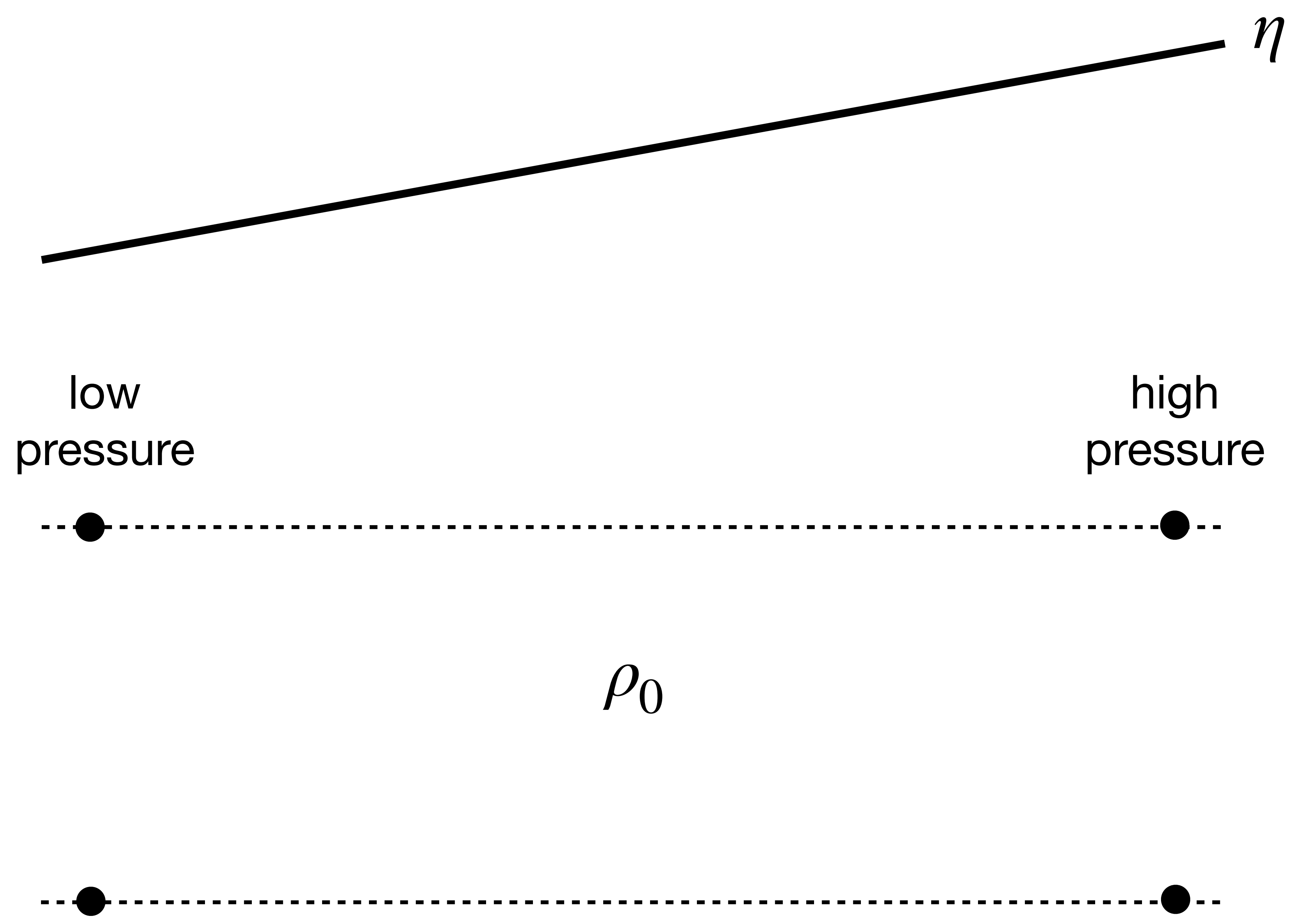
- The pressure gradient is at one z-level is the same at every z-level.

- If the pressure gradient is the same at every level, the geostrophic velocity is also the same at every level

- Geostrophic velocity is vertically uniform.

Geostrophy + constant density

Vertically uniform geostrophic flow



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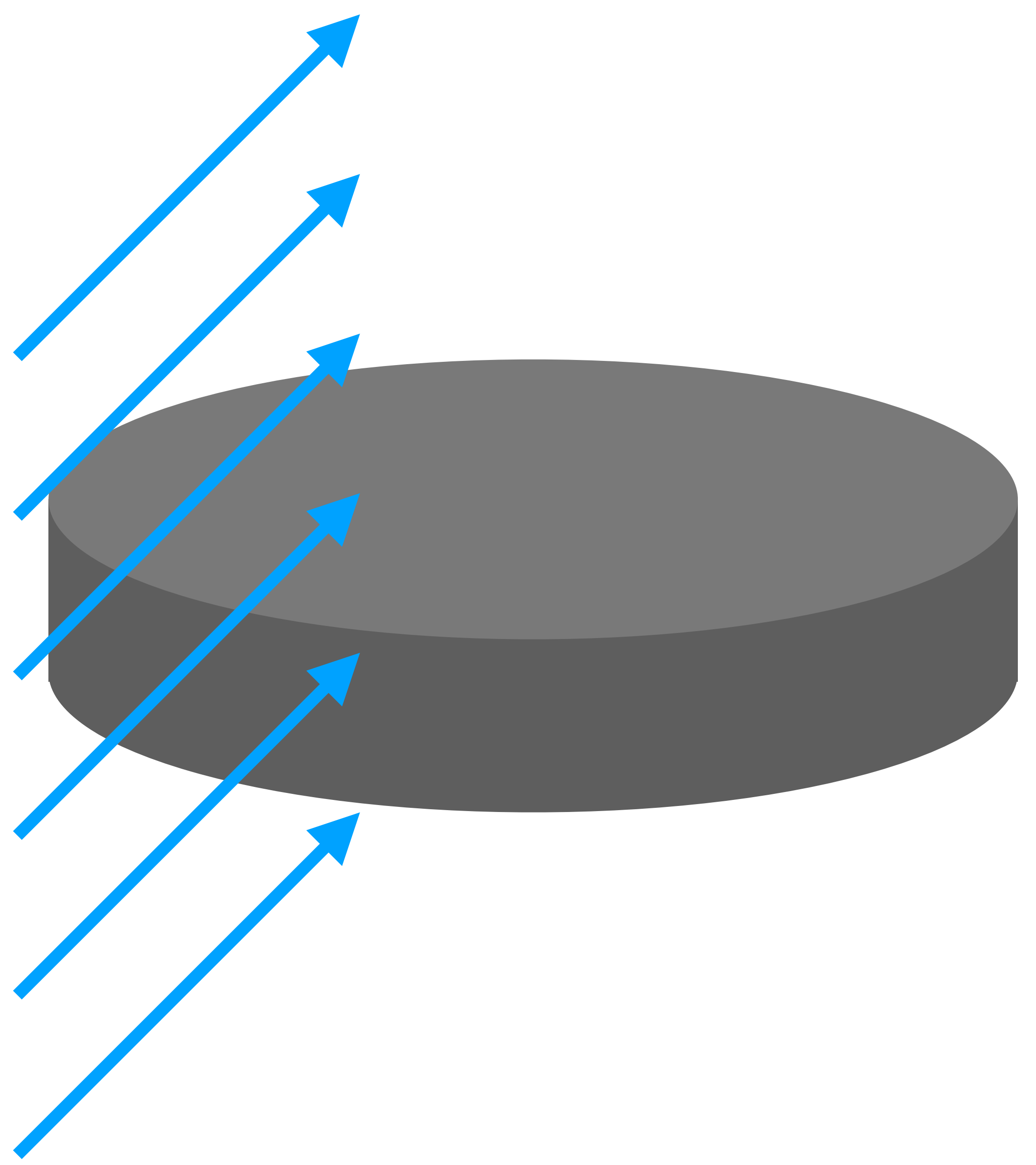
Geostrophy + constant density

Taylor columns

- Geostrophic velocity is vertically uniform in fluid with a constant density.
- Fluid behaves like it is **locked together in the vertical**
  - Imagine a bunch of vertical chopsticks moving around and past each other, but always remaining vertical.
- What happens when such a flow encounters a partial obstacle?



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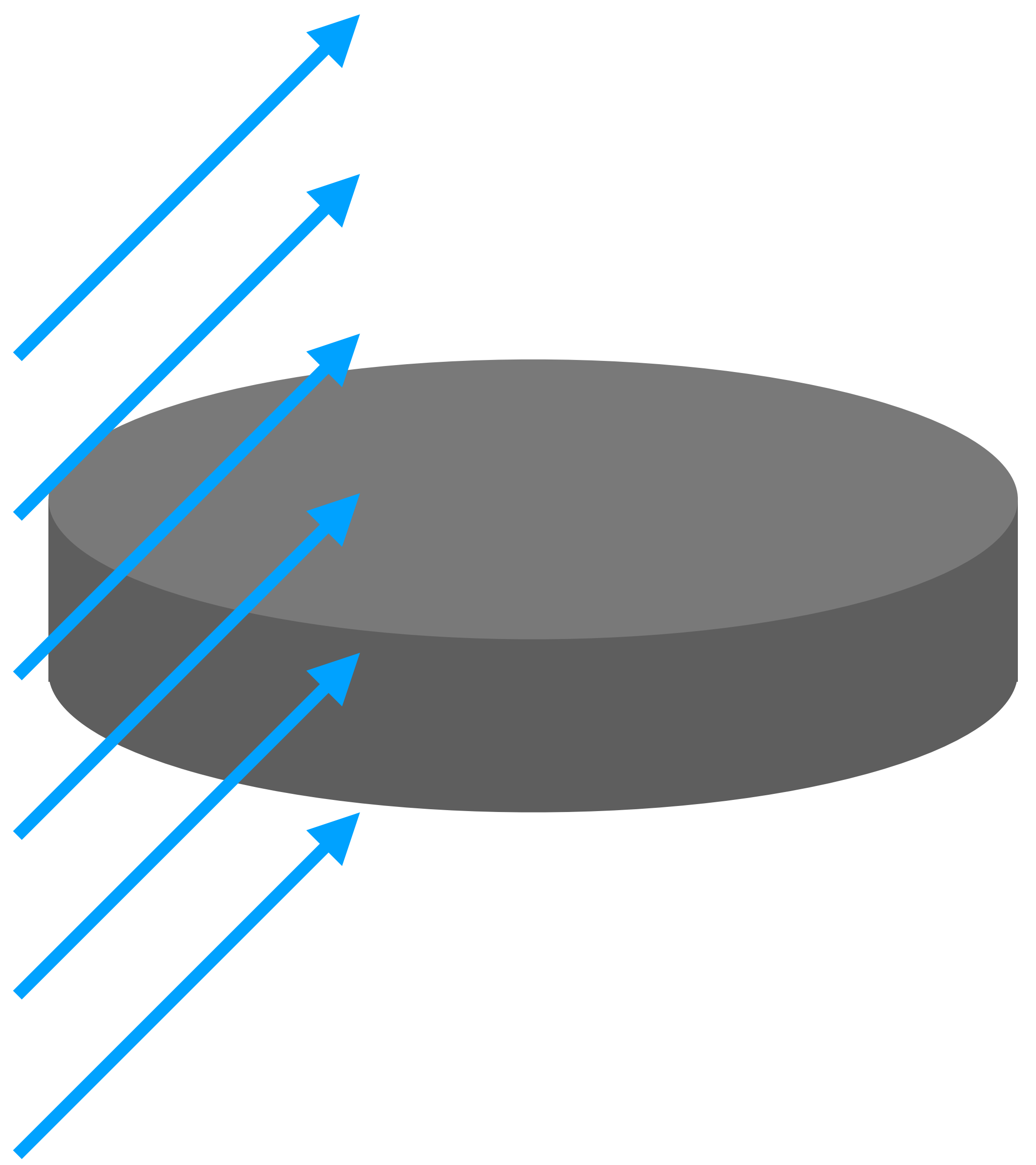
  - No flow over; no flow off.
  - Fluid trapped over the puck is a **Taylor column**.



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Mean drifter velocity in ACC near Kerguelen Plateau