Homework 4

1. Planetary vorticity is the portion of a water parcel's total vorticity that is due to Earth's rotation. In ocean dynamics, it is the local vertical component of planetary vorticity that matters. The local vertical component of planetary vorticity is the Coriolis parameter, $f = 2|\vec{\Omega}| \sin \theta$, where $\vec{\Omega}$ is Earth's angular velocity, and θ is latitude.

   Relative vorticity is the portion of total vorticity that is relative to Earth's rotating reference frame. It is calculated by taking the curl of the velocity field. In this class, it is the local vertical component of relative vorticity that matters, which can be calculated as $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$.

2. $\frac{D}{Dt}\left(\frac{\zeta + f}{H}\right) = 0$

a) Increase in ζ $\longrightarrow$ decrease in f, and/or increase in H

b) Increase f $\longrightarrow$ decrease in ζ, and/or increase in H

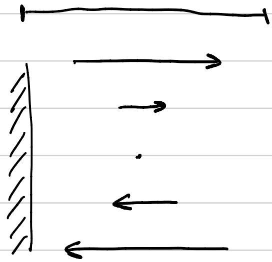
c) Decrease H $\longrightarrow$ decrease in ζ, and/or decrease in H

3. Over Northern Hemisphere subtropical gyres, the winds over the ocean change from westward (i.e., trades) to eastward (i.e., westerlies) toward the pole. Over the upper tens of meters of the ocean (i.e., the Ekman layer) the bulk movement of water is 90° to the right of the wind direction. As a result, the bulk movement of ocean water is convergent within tens of meters of the surface. Mass conservation requires that convergence in the surface region be balanced by squashing and flattening of water columns in the ocean interior. On its own, the flattening of water columns increases angular momentum. To conserve angular momentum the flattened colums move southward to decrease angular momentum due to Earth's rotation. In order to conserve mass, the southward motion over the entire east-west extent of the gyre requires concentrated northward motion in the western boundary region. The same arguments can be made for Southern Hemisphere subtropical gyres, but the boundary current will be southward, or poleward in both hemispheres.

4. If wind stress doubles over the N. Atlantic, but the pattern stays the same, then each aspect of large-scale ocean circulation would also double. Ekman transport would be twice as large. Wind-stress curl — and hence Sverdrup transport — would be twice as large. The western boundary current would also double in strength in response to doubling of Sverdrup transport.

5. In the ocean interior away from the surface, potential vorticity f/H is conserved. In the real ocean, f increases with latitude so that a f/H increases from south to north over a region of the ocean with constant H. In the tank, f is constant. We simulate increasing f/H with latitude by sloping the bottom of the tank up from "south" to "north".

6. $\Delta x = 1 \times 10^{7}\,m$



$U_w^x = 7\ m/s$, $\tau_w^x = 7 \times 10^{-2}\ N/m^2$ @ 30° N

$U_w^x = 0\ m/s$, $\tau_w^x = 0\ N/m^2$ @ 25° N

$U_w^x = -7\ m/s$, $\tau_w^x = -7 \times 10^{-2}\ N/m^2$ @ 20° N

$$\left.\begin{array}{ll} 20°N: & f = 5 \times 10^{-5}\ \frac{1}{s} \\ 25°N: & f = 6 \times 10^{-5}\ \frac{1}{s} \\ 30°N: & f = 7 \times 10^{-5}\ 1/s \end{array}\right\} \quad \beta = 2 \times 10^{-11}\ \frac{1}{ms}$$

a) Ekman transport, $V_e = -\dfrac{\tau_w^x}{\rho_0 f}$, is transport per unit width.

@ 20°N : $$V_e = -\frac{-7 \times 10^{-2}\ \frac{kg\,m}{s^2} \cdot \frac{1}{m^2}}{(1 \times 10^{3}\ kg/m^3)(5 \times 10^{-5}\ \frac{1}{s})} = 1.4\ m^2/s$$

$$V_e\,\Delta x = 1.4 \times 10^{7}\ m^3/s$$

$$= \underline{14\ Sv}\ \text{(to the north)}$$

@ 25°N : $V_e = 0\ m^2/s \Rightarrow V_e\,\Delta x = \underline{0\ Sv}$

@ 30°N : Swap $f = 6 \times 10^{-5}\ \frac{1}{s}$ in calculation for 20°N and change sign for τ_w^x.

$$V_e\,\Delta x \approx -12\ Sv\ \text{(to the south)}$$

b) Convergent. Downward. It is consistent with wind stress curl, because negative wind-stress curl (such as prescribed here) is balanced by downward Ekman pumping.

c) To the south. Downward Ekman pumping vertically squashes water parcels, which must be balanced by southward motion to conserve f/H.

d) Assume β is constant with latitude. Since wind stress changes linearly with latitude, wind-stress curl is constant. So, the answer is the same for all latitudes.

$$V = -\frac{1}{\rho_0 \beta}\frac{\partial \tau_w^x}{\partial y}, \qquad \frac{\partial \tau_w^x}{\partial y} \approx \frac{1.4 \times 10^{-1}\ N/m^2}{1 \times 10^6\ m}$$

$$= 1.4 \times 10^{-7}\ \frac{N}{m^3}$$

$$\downarrow$$

$$\frac{kg\,\cancel{m}}{s^2}\cdot\frac{1}{m^{\cancel{3}\,2}}$$

$$V \approx -\frac{1.4 \times 10^{-7}\ \frac{\cancel{kg}}{s^2\,\cancel{m^2}}}{(1 \times 10^3\ \cancel{kg/m^3})(2 \times 10^{-11}\ \frac{1}{ms})}$$

$$= -7\ m^2/s, \text{ to the south}$$

Across the ocean basin of width 1×10^7 m, total transport is $7 \times 10^7\ m^3/s$ or $\underline{\underline{70\ Sv}}$ to the south.

e) For ocean depth of 1×10^3 m, velocity is

$$v = \frac{-7 \ m^2/s}{1 \times 10^3 \ m} = -7 \times 10^{-3} \ m/s = -7 \ mm/s$$

f) Sverdrup transport is geostrophic plus Ekman, so $V_g = V_{tot} - V_e$.

@ 20° N : $V_g = -70 - 14 = -84$ Sv
@ 25° N : $ = -70 + 0 = -70$ Sv
@ 30° N : $ = -70 + 12 = -58$ Sv

all to the south

g) Western boundary current transport balances Sverdrup transport, so is 70 Sv to the north at all latitudes.

h) $$v = \frac{7 \times 10^7 \ m^3/s}{(1 \times 10^3 \ m)(1 \times 10^5 \ m)} = 0.7 \ m/s$$ to the north

7. AABW is fresher and colder than NADW. AABW is formed when very cold water upwelled in the Weddell Sea becomes saltier during sea ice formation. NADW forms when salty surface water in the subpolar N. Atlantic is cooled during winter and becomes dense enough to sink.

8. a) NADW

b) CW

c) NADW

d) CW

e) AABW

f) AABW

g) AABW

h) CW

i) NADW

j) NADW