Homework 2

1. a) Assuming the near surface ocean has uniform density, ρ, then the difference in pressure between the surface, η, and some depth, z, is

$$p(\eta) - p(z) = -g\rho(\eta - z)$$

If $p(z = 100\text{ m})$ is constant, then the change in surface pressure, $p(\eta)$, must be balanced by a change in η.

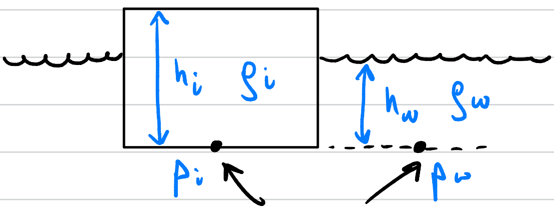
$$\Delta p(\eta) = -g\rho\Delta\eta \implies \Delta\eta = -\frac{\Delta p(\eta)}{g\rho}$$

$$\Delta\eta = -\frac{-10^2\ \text{N/m}^2}{(10\ \text{m/s}^2)(10^3\ \text{kg/m}^3)} = 10^{-2}\ \text{m} = \boxed{1\ \text{cm}}$$

b) In part a) we found 1 mb surface pressure decrease is balanced by 1 cm of water level increase.

A 100 mb decrease in surface pressure due to a hurricane contributes $\boxed{1\ \text{m}}$ of surge.

2. Hydrostatic pressure underneath the iceberg must be the same as pressure at the same depth in ice-free water.



hydrostatic pressure must be equal in these two locations

Remember that hydrostatic pressure beneath a layer of constant density is the weight of what's above.

$p(z) = g\rho(\eta - z) = g\rho h$ For ice : $p_i = g\rho_i h_i$

For water : $p_w = g\rho_w h_w$

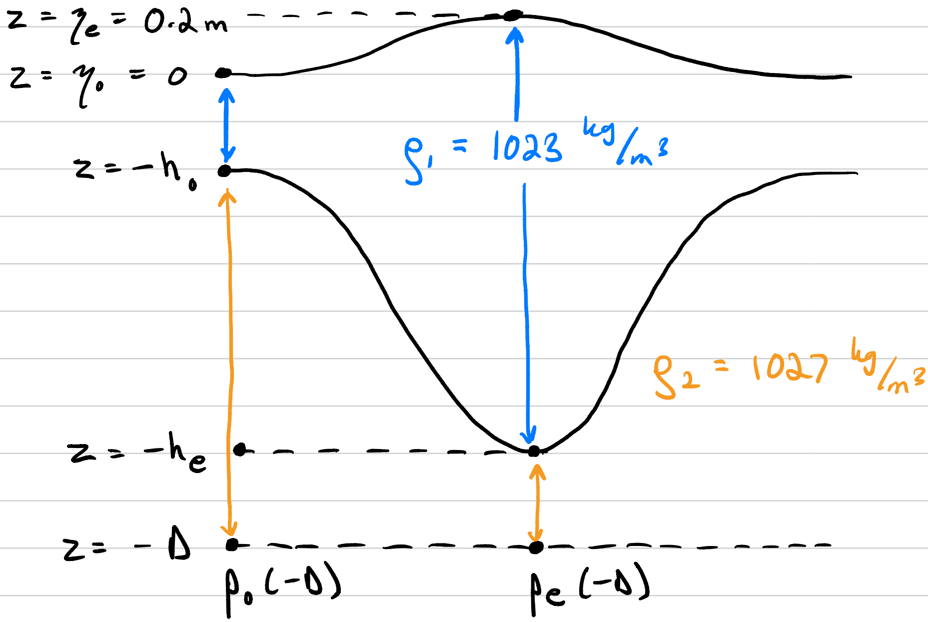
Since $p_w = p_i$, then $\rho_i h_i = \rho_w h_w$, or

$$\frac{h_w}{h_i} = \frac{\rho_i}{\rho_w}$$

The % of the iceberg's height above the water is

$$\frac{h_i - h_w}{h_i} = 1 - \frac{\rho_i}{\rho_w} = 1 - \frac{900 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = 0.1 = \boxed{10\%}$$

3. a)



b) Pressure in the lower layer @ center of the eddy equals pressure @ the periphery.

$$p_0(-\Delta) = p_e(-\Delta)$$

Use integrated form of hydrostatic balance to calculate $p_0(-\Delta)$ and $p_e(-\Delta)$.

$$p_0(-\Delta) = g\rho_1(\eta_0 + h_0) + g\rho_2(-h_0 + \Delta)$$

$$p_e(-\Delta) = g\rho_1(\eta_e + h_e) + g\rho_2(-h_e + \Delta)$$

Set these equal and get $h_e - h_0$ (the difference in layer in the eddy vs. outside.

Terms with Δ cancel, g cancels from all terms, and $\eta_0 = 0$. We are left with

$$\rho_1 h_0 - \rho_2 h_0 = \rho_1 \eta_e + \rho_1 h_e - \rho_2 h_e$$

$$(\rho_1 - \rho_2) h_0 = \rho_1 \eta_e + (\rho_1 - \rho_2) h_e$$

$$(\rho_1 - \rho_2)(h_0 - h_e) = \rho_1 \eta_e$$

$$h_e - h_0 = \frac{\rho_1 \eta_e}{\rho_2 - \rho_1}$$

$$h_e - h_0 = \frac{(1027 \text{ kg/m}^3)(0.2 \text{ m})}{1027 \text{ kg/m}^3 - 1023 \text{ kg/m}^3}$$

$$\approx \frac{(10^{\cancel{3}\,2} \text{ kg/m}^3)(\cancel{2} \times 10^{\cancel{-1}} \text{ m})}{\cancel{4} \text{ kg/m}^3}$$

$$\approx 0.5 \times 10^2 \text{ m} = \boxed{50 \text{ m}}$$

The layer depth within the eddy is 50 m lower than the depth in surrounding water.

c) Changing the average layer thickness is equivalent to adding a uniform density slab, which would not affect the difference in layer thickness across the eddy.

4. $\Omega = |\vec{\Omega}| \approx 7.3 \times 10^{-5}$ rad/s , $R_\oplus = 6400$ km

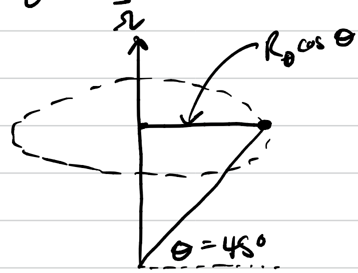
a) Centrifugal force per unit mass has magnitude

$$|\vec{\Omega} \times (\vec{\Omega} \times \vec{x})| = \Omega^2 R_\oplus \text{ @ equator}$$
$$= (7.3 \times 10^{-5} \text{ rad/s})^2 (6400 \text{ km})$$
$$= 0.03 \text{ m/s}^2$$

directed radially outward from Earth's rotation axis.

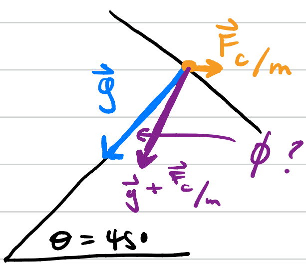
b) Gravitational acceleration $g \approx 10$ m/s^2 , so centrifugal force is roughly 0.3 % as large as gravitational acceleration at the equator.

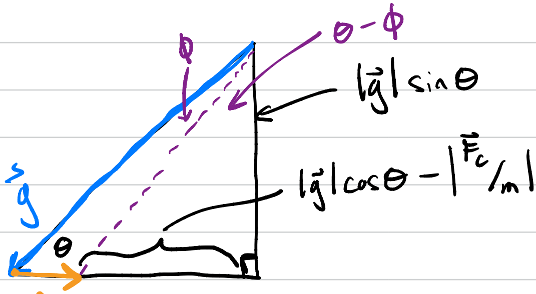
c) Centrifugal force per unit mass at 45°N has magnitude



$$\Omega^2 R_\oplus \cos\theta = (0.03 \text{ m/s}^2)(0.7)$$
$$\approx 0.02 \text{ m/s}^2$$

d) Gravity points toward Earth's center of mass :





$|\vec{F}_c/m| = \Omega^2 R_\oplus \cos\theta$ from c)

→ next page

$$\tan(\theta - \phi) = \frac{|\vec{g}|\cos\theta - |\vec{F_c}/m|}{|\vec{g}|\sin\theta}$$

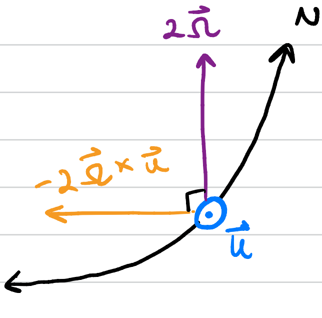
$$= 1 - \frac{0.02\ m/s^2}{(10\ m/s^2)(\sin 45°)} \quad \text{← from part c)}$$

$$= 0.997$$

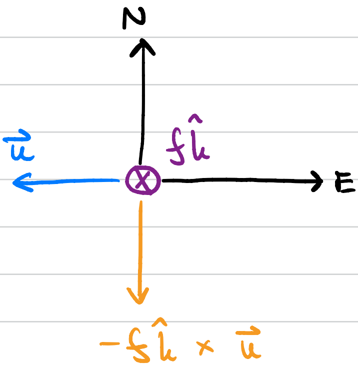
$$\phi = 45° - \tan^{-1}(0.997)$$

$$\boxed{\phi \approx 0.09°}$$

5. a)

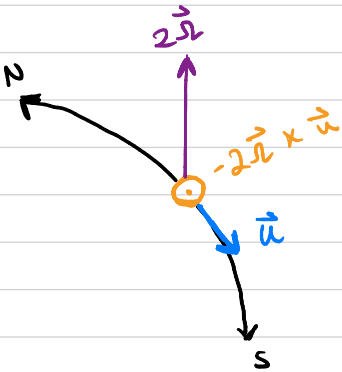


$2\vec{\Omega}$
N
$-2\vec{\Omega} \times \vec{u}$
$\vec{u}$
S

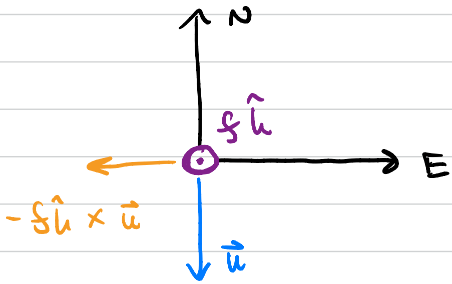


N
$f\hat{k}$
$\vec{u}$
E
$-f\hat{k} \times \vec{u}$

b)



$2\vec{\Omega}$
N
$-2\vec{\Omega} \times \vec{u}$
$\vec{u}$
S



N
$f\hat{k}$
E
$-f\hat{k} \times \vec{u}$
$\vec{u}$

6. a) Coriolis force per unit mass is

$$\vec{F}_c = fv\hat{i} - fu\hat{j}.$$

In this case (eastward velocity) $v = 0$, so

$$\vec{F}_c = -fu\hat{j}.$$

Remember $f = 2\Omega \sin\theta \approx 1 \times 10^{-4}\ ^1/_s$ @ $45°N$.

$$\frac{\vec{F}_c}{m} = -(1 \times 10^{-4}\ ^1/_s)(2 \times 10^1\ ^m/_s)\hat{j}$$

$$= \boxed{-2 \times 10^{-3}\ ^m/_{s^2}\ \hat{j}}$$

Negative in the $\hat{j}$ direction is to the $\boxed{\text{south}}$.

b) $t = {}^{60m}/_{20\ m/s} = 3\ s$, $a = {}^{\vec{F}_c}/_m$

$$d = \frac{1}{2}at^2 = \frac{1}{2}(2 \times 10^{-3}\ ^m/_{s^2})(9\ s^2)$$

$\approx 1 \times 10^{-2}$ m, or $\boxed{1\ \text{cm}}$ to the south.

7. a) Ekman balance is the balance between the coriolis and friction terms.

b) No. wind stress is applied at the surface. It is a boundary condition, not a body force.

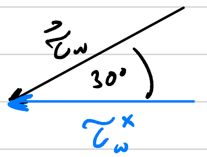
c) Ekman transport does not depend on the depth of the Ekman layer, because as momentum diffuses downward, the stress eventually decays to zero. Adding up (i.e. integrating) the stress divergence over the Ekman layer with zero stress at its base leaves only a dependence on the surface value (i.e., wind stress). The details of the momentum diffusion and vertical structure of the velocity do not matter!

8\. a) $\vec{\tau}_w = C_d \rho_a |\vec{U}_{10}| \vec{U}_{10}$

For $|\vec{U}_{10}| = 7 \ m/s$, $|\vec{\tau}_w| = C_d \rho_a |\vec{U}_{10}|^2$

$|\vec{\tau}_w| = (1.2 \times 10^{-3})(1.2 \ kg/m^3)(7 \ m/s)^2$

$\approx \boxed{7 \times 10^{-2} \ N/m^2}$

b) 

$\tau_w^x = -|\vec{\tau}_w| \cos 30°$

$\approx \boxed{-6 \times 10^{-2} \ N/m^2}$

c) $V_e = -\dfrac{\tau_w^x}{\rho_o f} = -\dfrac{-6 \times 10^{-2} \ N/m^2}{(10^3 \ kg/m^3)(5 \times 10^{-5} \ 1/s)}$

$= \boxed{1.2 \ m^2/s}$

d) $v_e = \dfrac{V_e}{D} = \dfrac{1.2 \ m^2/s}{40 \ m} = 3 \times 10^{-2} \ m/s = \boxed{3 \ cm/s}$

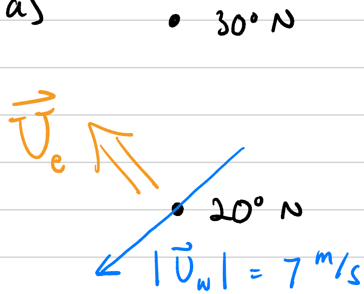
e) $100°$ longitude $\approx 1 \times 10^7 \ m = \Delta x$

$V_e \Delta x = 1.2 \times 10^7 \ m^3/s = \boxed{12 \ Sv}$

f) $|\vec{\tau}_w| \sim |\vec{U}_w|^2 \Rightarrow$ halving the wind speed results in wind stress magnitude that is 1/4 the original. Thus, volume transport would be reduced to $\boxed{3 \ Sv}$.

From 8a

9. a)



@ 20° N : $|\vec{\tau}_w| = 7 \times 10^{-2} \, N/m^2$

$$|\vec{U}_e| = -\frac{|\vec{\tau}_w|}{\rho_0 f}$$

$$= \frac{7 \times 10^{-2} \, N/m^2}{(10^3 \, kg/m^3)(5 \times 10^{-5} \, 1/s)}$$

$$= \boxed{1.4 \, m^2/s} \text{ to the } \boxed{NW}$$

@ 30° N : $|\vec{\tau}_w| = 0 \Rightarrow |\vec{U}_e| = \boxed{0 \, m^2/s}$

b) @ 20° N : $\vec{U}_e = U_e \hat{i} + V_e \hat{j}$

$$U_e = -|\vec{U}_e| \sin \pi/4 = \boxed{-1 \, m^2/s} \text{, west}$$

$$V_e = |\vec{U}_e| \cos \pi/4 = \boxed{1 \, m^2/s} \text{, north}$$

c) There is convergence in the meridional direction of

$$\frac{\Delta V_e}{\Delta y} = \frac{-1 \, m^2/s}{10^6 \, m} = \boxed{-10^{-6} \, m/s}$$

We cannot say anything about zonal divergence.