Homework 1

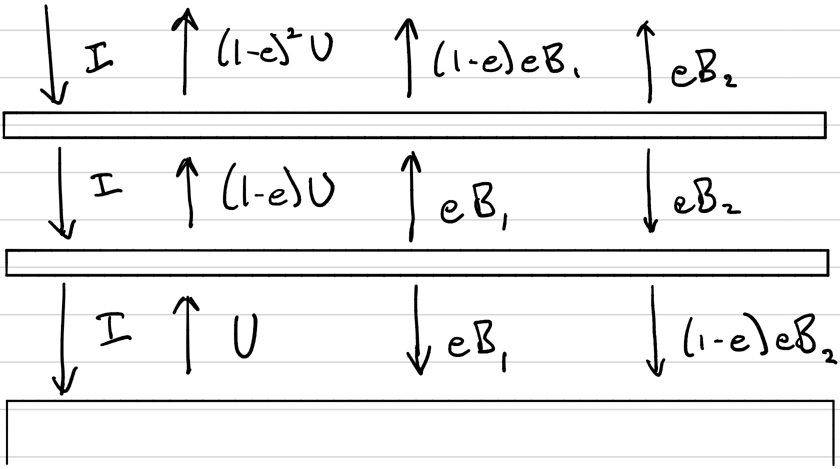
1. a) $\frac{\partial S}{\partial y}$

   b) $\frac{DT}{Dt}$

   c) $\nabla \cdot \vec{u}$

   d) $\nabla \cdot \vec{u} = 0$

3. a)



b)
$$0 = -I + (1-e)^2 U + (1-e)eB_1 + eB_2$$
$$0 = -I + (1-e)U + eB_1 - eB_2$$
$$0 = -I + U - eB_1 - (1-e)eB_2$$

Add first two equations to eliminate B_2:
$$0 = -2I + (1-e)(2-e)U + e(2-e)B_1$$

Multiply second equation by $-(1-e)$ and add to third equation to eliminate B_2:
$$0 = -eI + e(2-e)U - e(2-e)B_1$$

Add these two new equations to eliminate B_1:
$$0 = -(2+e)I + (2-e)U$$

Solve for U:
$$U = \frac{2+e}{2-e} I$$

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c) In class we used $I = 240 \, W/m^2$, $e = 0.9$.
This gives $U = 633 \, W/m^2$

$$T_g = \left(\frac{U}{\sigma}\right)^{1/4} = 325 \, K = 52°C$$

Way too hot! What e gives $T_g = 14°C$?

Solving for e gives

$$e = 2\left(\frac{\sigma T^4 - I}{\sigma T^4 + I}\right).$$

Substituting $T = 14°C = 287 \, K$ gives $e \approx 0.5$.

4. a) $\alpha_r = 5.4 \ W/m^2$

$X_{CO_2}(t_0) = 315$ ppm for $t_0 = 1958$

$X_{CO_2}(t) = 415$ ppm for $t = 2020$

$\Delta H = 1.5 \ W/m^2$

b) Approximate the average extra heating as

$\overline{\Delta H} \approx \frac{1}{2}\Delta H = 0.75 \ W/m^2$

The total time, Δt, is 63 years $\approx 2 \times 10^9$ sec.

The upper ocean has depth, $D = 500$ m, and surface area A.

The temperature change, ΔT, is then

$$\Delta T = \frac{\overbrace{\overline{\Delta H}\,\Delta t\,A}^{\text{Energy in J}}}{\underbrace{DA\rho}_{\text{mass in kg}}\,c_p} = \frac{\overline{\Delta H}\,\Delta t}{D\rho c_p}$$

$$\Delta T = 0.75\,°C$$

5. $Q_{LH} = 175\ W/m^2$, $\Delta t \approx 3 \times 10^7$ sec

Latent heat of vaporization : $L_v = 2.5 \times 10^6\ J/kg$

Rate of evaporation in $\frac{kg}{m^2 s}$ is $E = \frac{Q_{LH}}{L_v}$.

Over one year , $E \Delta t = \frac{Q_{LH}\, \Delta t}{L_v} = \underline{2 \times 10^3\ \frac{kg}{m^2}}$

Thickness of layer with that mass and area :

$$\underbrace{\frac{2 \times 10^3\ kg/m^2}{1 \times 10^3\ kg/m^3}}_{\rho} = \underline{2m}$$

6. When $|\vec{U}_{10}|$ increases :

- $\vec{\tau}$ increases as square of the wind speed.
- Q_{SH} increases .
- E increases .
- Q_{LH} increases .
- B increases with E having a dual effect on heat and freshwater fluxes.

7. When T increases ;

- Q_{SH} increases — most efficient when sea is warmer than air
- E increases exponentially as specific humidity increases exponentially with T.
- Q_{LH} increases exponentially in proportion to E.
- Q_{LW} decreases due to increased backradiation from near-surface evaporation despite greater outgoing LW from increased T.
- B increases due to increases in heat flux and evaporation .

8\. a) $\vec{\tau} = C_d \rho_a |\vec{U}_{10}| \vec{U}_{10}$ $C_d \approx 1.2 \times 10^{-3}$

$\rho_a \approx 1.2\ kg/m^3$

$|\vec{\tau}| = 7 \times 10^{-2}\ N/m^2$

b) $m = \frac{F_g}{g} = \frac{7 \times 10^{-2}\ N}{10\ m/s^2} = 7 \times 10^{-3}\ kg$

c) $V = \frac{m}{\rho} = 7 \times 10^{-6}\ m^3$

d) $\delta = \frac{V}{A} = 7 \times 10^{-6}\ m$ or 7 microns

e) Gravity is strong! The tradewinds apply a force per unit area on the ocean that is equivalent to the downward gravitational pressure of an extremely thin layer of water. It is important to remember, however, that these forces operate in orthogonal directions.

9. a) Molecular diffusion is <u>very</u> slow over the vast spatial scales we consider in the ocean. Eddy diffusion approximates mixing at macro scales, which are relevant for ocean observations and models.

b) Eddy diffusion occurs via turbulent motions of water parcels or "blobs". The fluid properties (heat, salt, etc.) are conserved within each blob, i.e., each blob retains its properties as it moves. Thus, eddy diffusivity is a property of the flow field, not a characteristic of the property in question. Molecular diffusion depends on the thermal properties of the substance being diffused.

c) Stable stratification in the vertical suppresses vertical turbulent eddy motion, leading to a lower vertical eddy diffusivity. Molecular diffusion occurs due to random thermal motion, which is isotropic.